convex risk

Pricing Insurance Risk: Theory and Practice

Stephen J. Mildenhall

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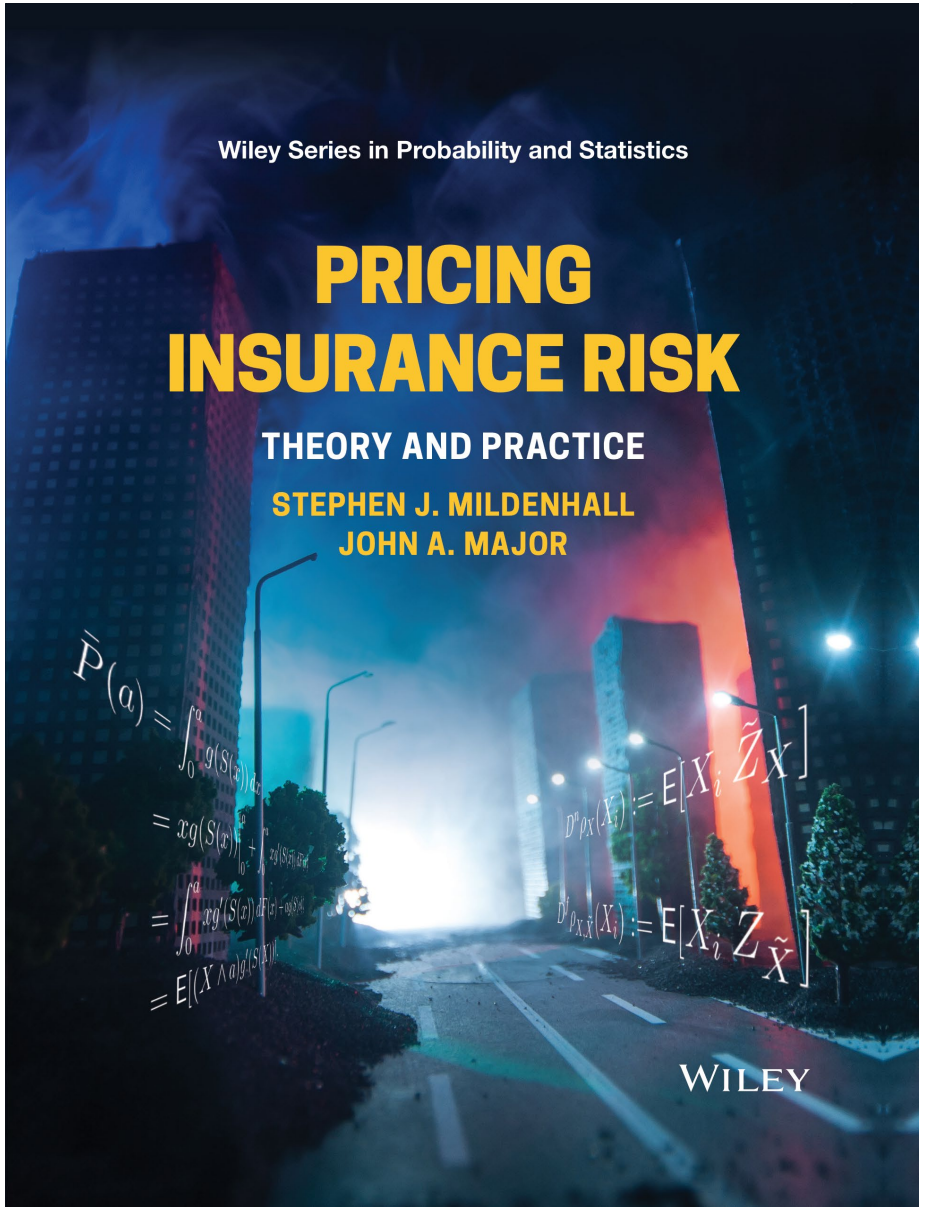

http://www.pricinginsurancerisk.com

- Case study exhibits
- Supplemental exhibits
- Presentations
- Errata

Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

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Poll: Which best describes the risk loads you expect to see for Cat relative to Non-cat exposed business?

A. Cat should have a significantly higher risk load than NonCat.
B. Cat should have a moderately higher risk load than NonCat.
C. The two Businesses should have the same risk load.
D. Cat should have a lower risk load than NonCat.

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Highlights

Adam Smith was ahead of his time. It takes two risk measures to price. The Cost of Capital (CoC) is not constant.

Modern finance provides a satisfying theoretical pricing model. But it doesn't quite work in practice and is hard to parameterize.

Allocate premium, not capital. Parameterize to financing and strategic decisions as well as premiums.

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Theory: The Answer

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Theory: ~~The~~ Answer

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Theory: An Answer

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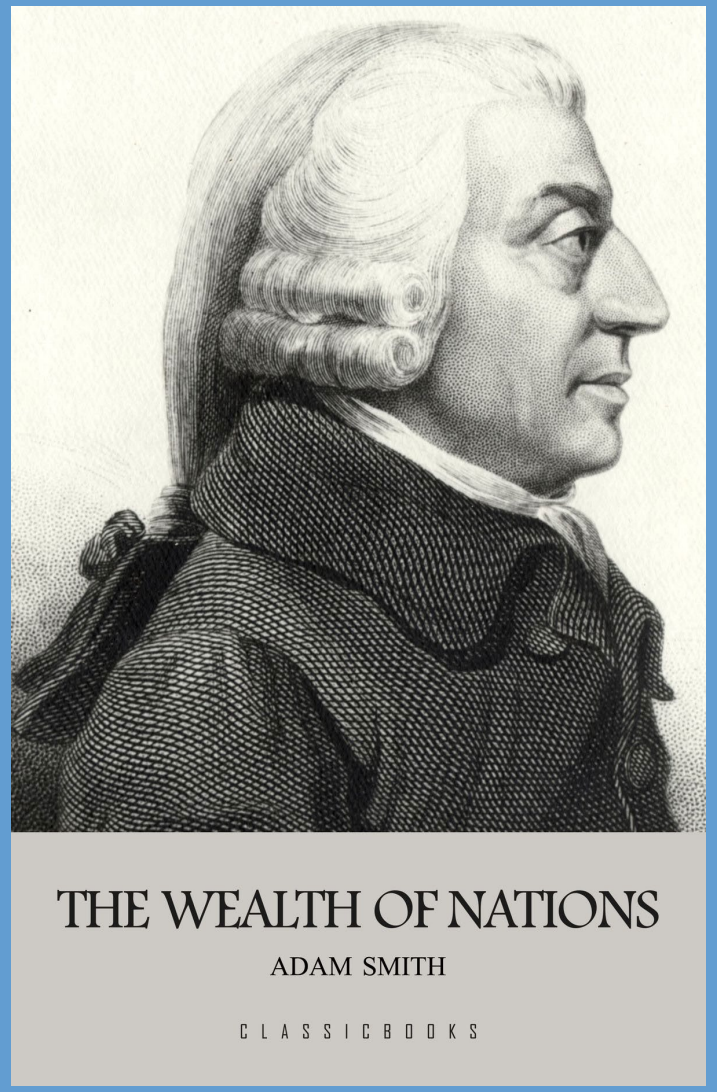

In order to make insurance a trade at all, the common premium must be sufficient to compensate the common losses, to pay the expense of management, and to **afford such a profit as might have been drawn from an equal capital employed in any common trade.**

Book 1, Ch X, Part I, 5th Edition, 1789

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Adam Smith Cost of Capital (CoC) Portfolio Pricing

Losses X plus two ingredients

- Amount of assets = a
- Cost of capital rate = ι

Observations

1. Assets = Premium + Capital
2. **Two** risk measures, a and ι
3. Asset amount exogenous, a(X)
4. Pricing $X \wedge a = \min(X, a)$, not X
5. Price related to capital structure

CoC premium formula

- P = (expected loss)
  + (cost of capital) x (amount)
  $= EL + \iota (a - P)$
  $= EL + d (a - EL)$ — EL + risk margin
  $= a - v (a - EL)$ — a – capital
  $= v\,EL + d\,a$ — $v\,E[\cdot] + d \max(\cdot)$

$v = 1 / (1 + \iota)$, risk discount factor

$d = \iota / (1 + \iota) = \iota v$, rate of risk discount

- Method-in-use in US rate filings

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Constant Cost of Capital? CoC *should* vary, but how?

"...the use of a company-wide cost of capital implicitly assumes that the new policy has the same risk-return characteristics as the firm as a whole. ...this assumption **may be questionable** in multiple line companies..."

Cummins, JRI 1990

- **C**CoC **across lines** originally resulted from estimation problems
- ...but extremely convenient: EVA, pricing becomes capital allocation
- Forgotten: credit yield curve shows CoC is manifestly not constant **across capital layers (priorities)**
- MM? Financing matters for allocation!

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Modern (Post-Coherent) Portfolio Pricing

Desirable properties

a) **Monotone**: X ≤ Y implies that ρ(X) ≤ ρ(Y)

b) **Sub-additive**: respects diversification: ρ(X + Y) ≤ ρ(X) + ρ(Y)

c) **Comonotonic additive**: no credit when no diversification. If outcomes X and Y imply same event order, then ρ(X + Y) = ρ(X) + ρ(Y)

d) **Law invariant**: ρ(X) depends only on the distribution of X; no categorical "line" CoC

A **spectral risk measure** (SRM) ρ(X) is defined by (a)-(d). It has four representations:

1. Weighted average of VaRs
2. Weighted average of TVaRs
3. Worst over a set of probability scenarios, max { E[XZ] | some Z }
4. Distorted expected value

$$\rho_g(X) := \int_0^\infty g(S_X(x))\,dx = \mathsf{E}[Xg'(S(X))]$$

for increasing, concave g

- **Natural allocation**: $\mathsf{E}[X_i\, g'(S(X))]$

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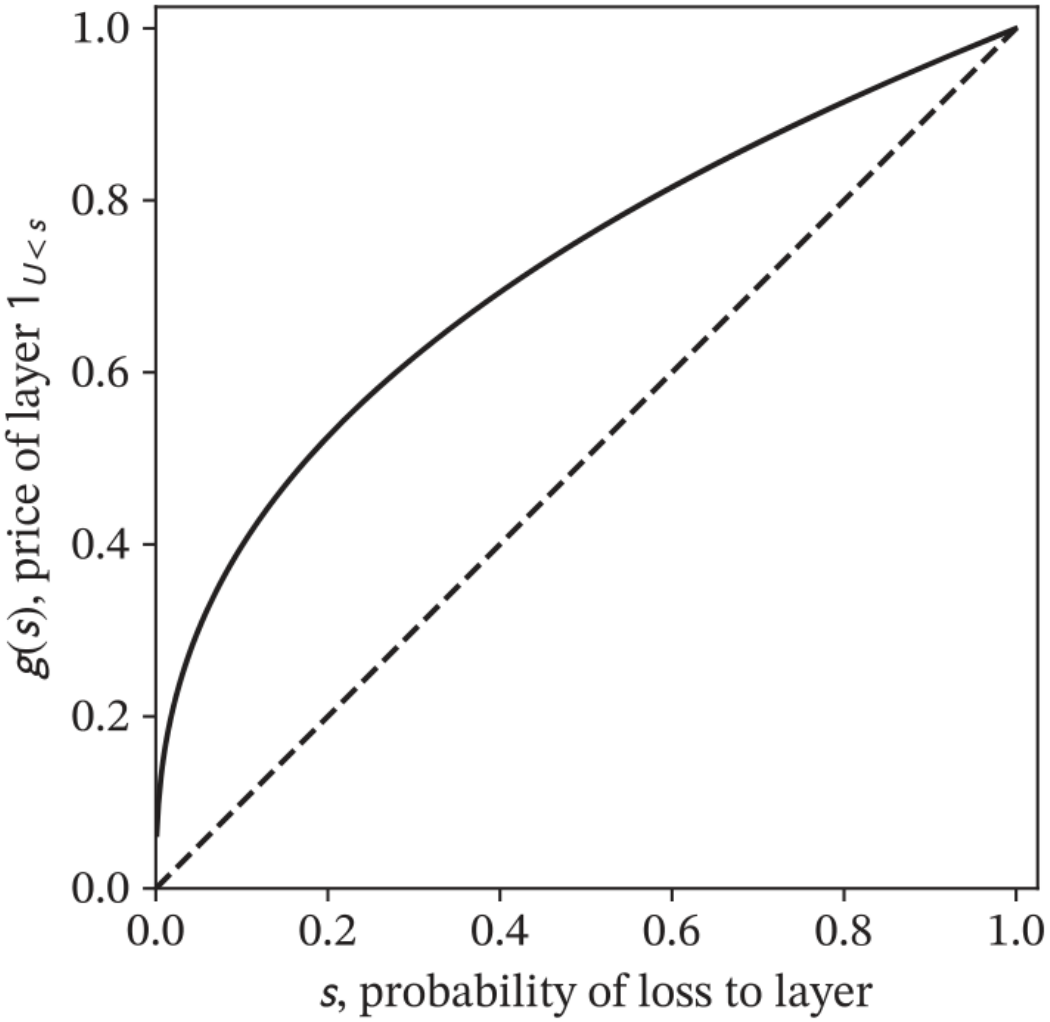
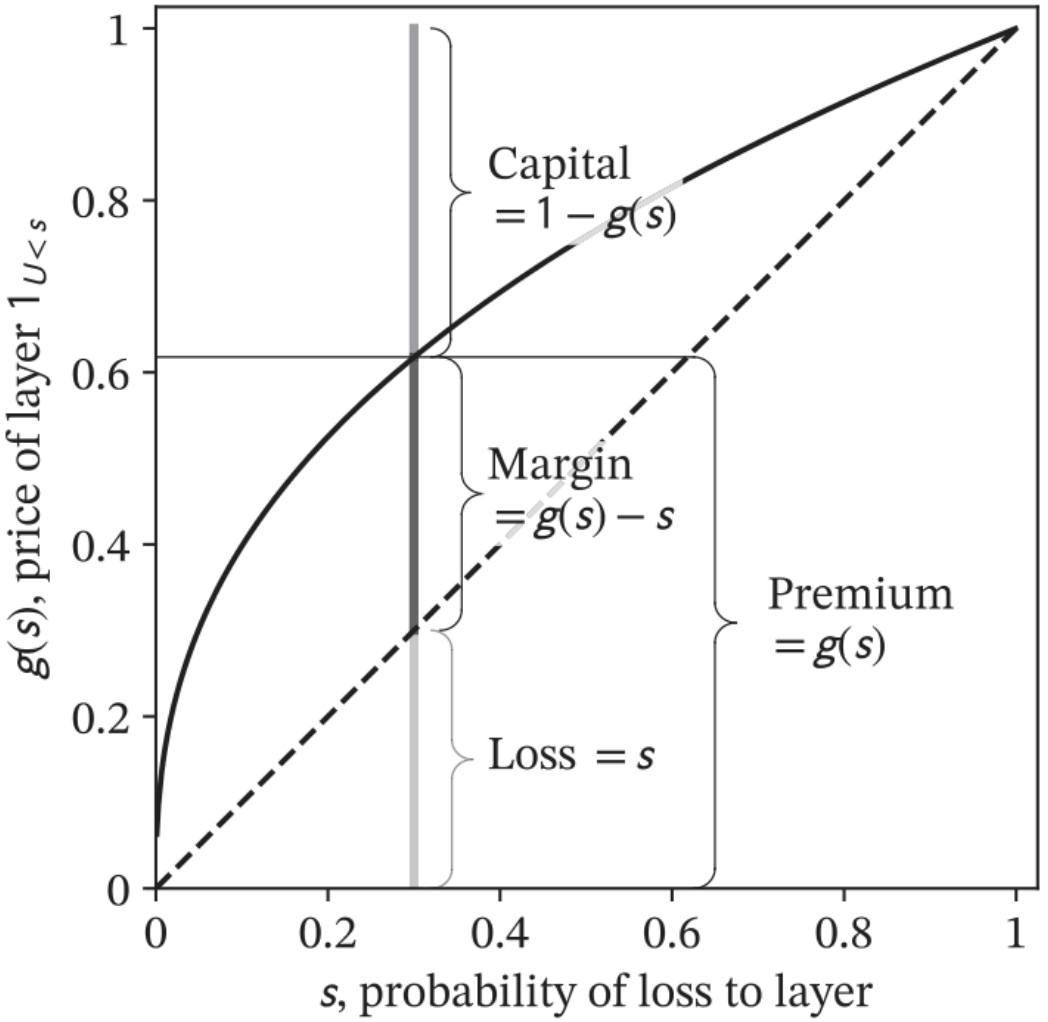
Distortion Function: g(s) = Ask Price for Bernoulli 0/1 Risk





Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

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Spectral Risk Measure Portfolio Pricing

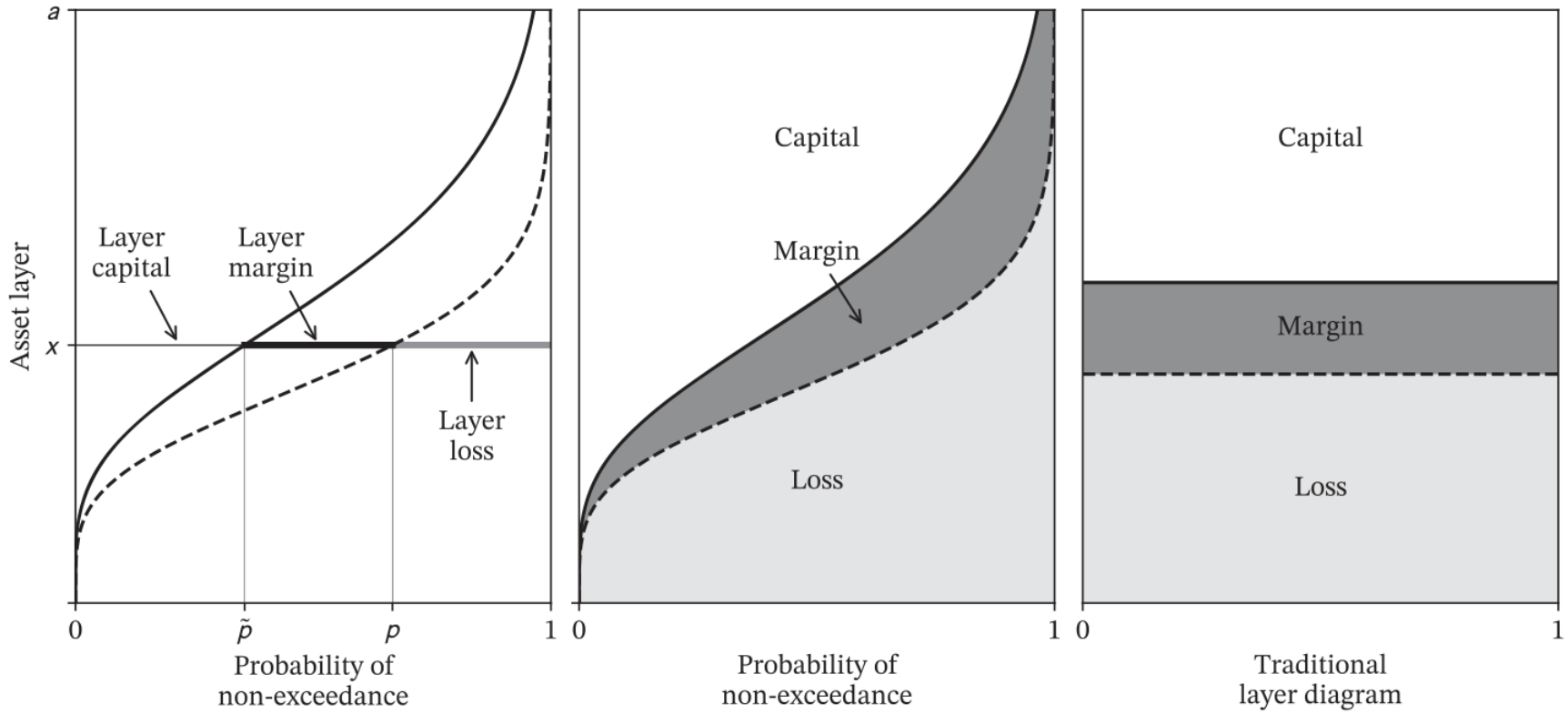

Figure 10.5 Continuum of risk sharing varying by layer of loss (dashed) and premium (solid): by layer (left), in total summed over layers (middle), and traditional (right). The total loss, margin, and capital areas are equal in the middle and right plots. Losses in the left and middle plots are Lee diagrams.

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Allocation: Marginal versus Natural

Marginal cost allocation

- Consistent with microeconomic optimization
- Euler, Billera and Heath
- Cost based: **insurer's** perspective

- **C**CoC makes cost ↔ capital
- Tasche, EVA, Meyers, Myers-Read

Natural allocation

- Intuitive and consistent with modern finance, risk adjusted probabilities
- Risk adjustment = g'(S(X))
- Benefit based: **insured's** perspective

- Known as co-measure in US
- Must know payments in default

…cannot always be equal

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Delbaen's Theorem (2000): Marginal Equals Natural...

If there is a **unique** density Z so that ρ(X) = E[XZ], then the marginal allocation equals the natural allocation

Comments

- Always have Z = g'(S(X))
- Qu: Are there others?
  Ans: No, iff q_X is increasing
- When Z not unique, choices include
  - **Linear** natural: Z ← E[Z | X]
  - **Lifted** natural: use X for X ∧ a

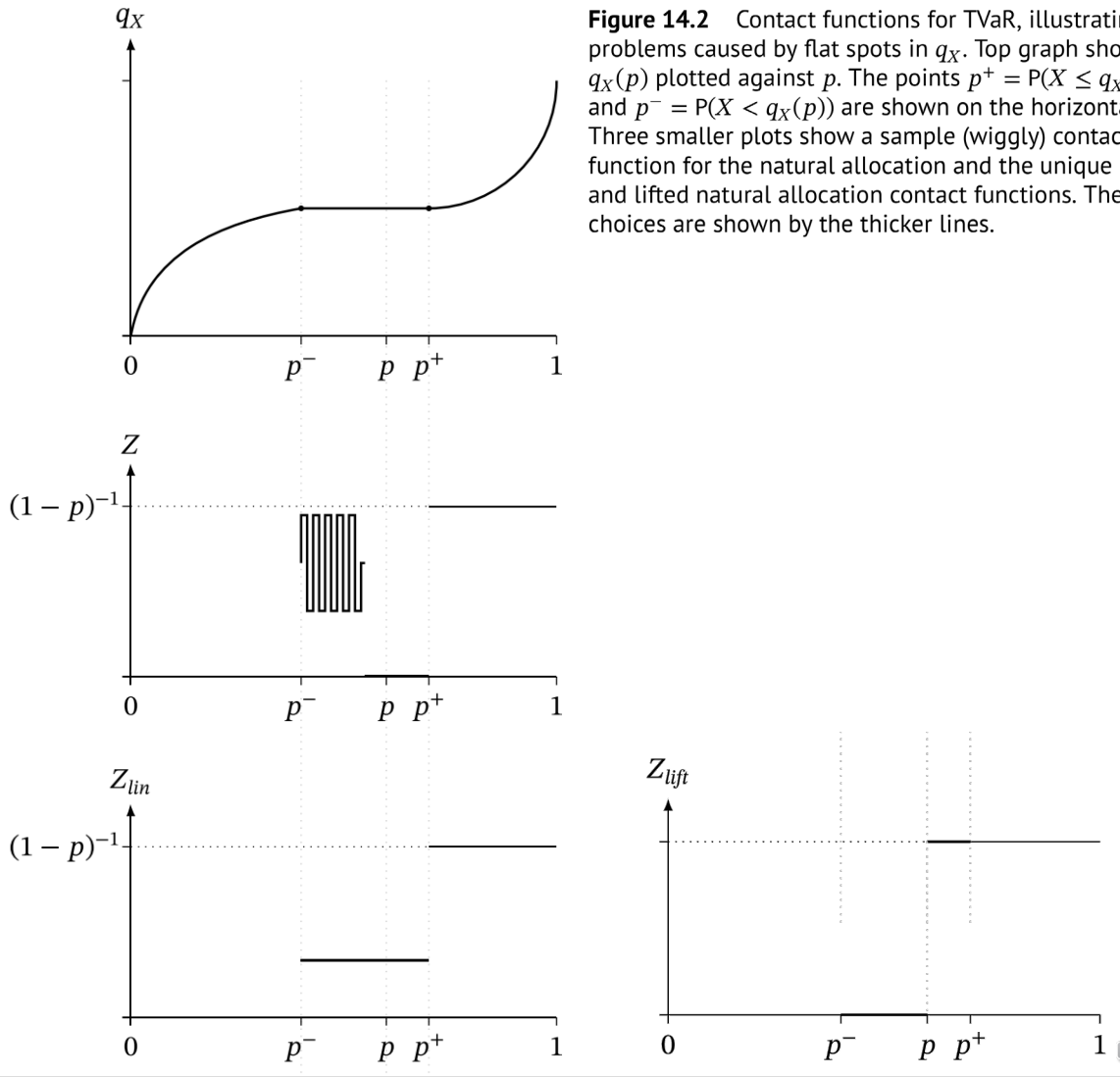

Figure 14.2 Contact functions for TVaR, illustrating the problems caused by flat spots in q_X. Top graph shows $q_X(p)$ plotted against p. The points $p^+ = \mathsf{P}(X \le q_X(p))$ and $p^- = \mathsf{P}(X < q_X(p))$ are shown on the horizontal axis. Three smaller plots show a sample (wiggly) contact function for the natural allocation and the unique linear and lifted natural allocation contact functions. The choices are shown by the thicker lines.

Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

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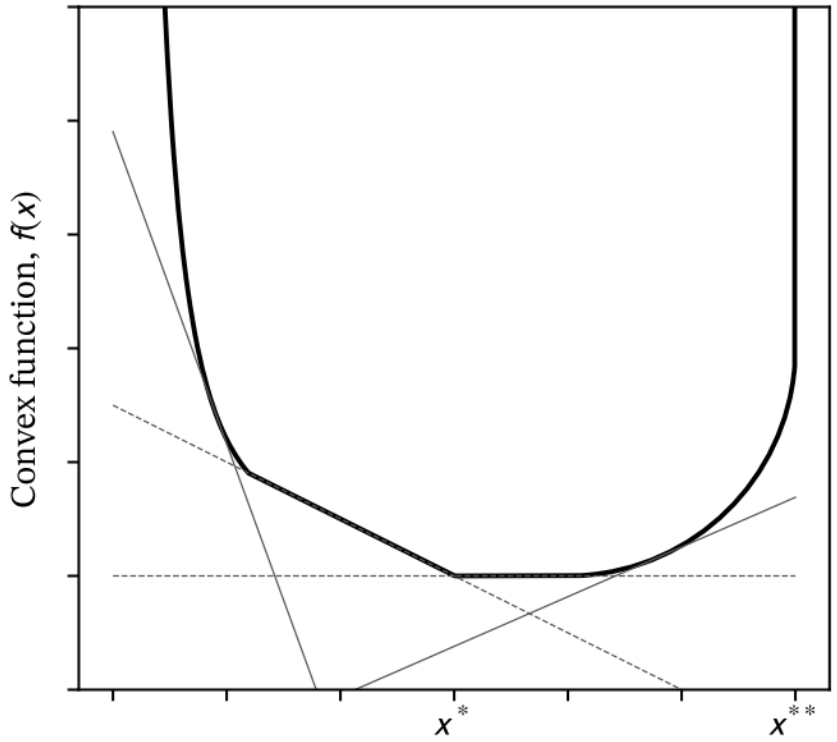
A Beautiful Theory

- Marginal = Natural unless
  - Convex ρ is not differentiable at X
  - Marginal left/right derivatives different
  - The order of writing matters
  - Default rules matters
  - **Working with X ∧ a and Pr(X>a) > 0**
- Delbaen: best answer to Venter, Major, Kreps (ASTIN 2006)
- Applies to homogeneous and inhomogeneous portfolios



Modern finance provides a satisfying theoretical pricing model. But it doesn't quite work in practice.

Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

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Practice: The Allocations

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To Apply SRMs Must Address…

1. Non-uniqueness caused by the flat spot in $X \wedge a$

- No definitive answer
- Linear and lifted allocations

2. Determine distortion function g

a) Directly

b) Indirectly

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2a) Estimating g Directly

- Interpretation: g(s) is the ask price to write the Bernoulli risk $1_{U<s}$, U uniform
- Comparables
  - Corporate bonds
  - Catastrophe bonds (perfect example InsCo!)
  - No data for s > 0.2

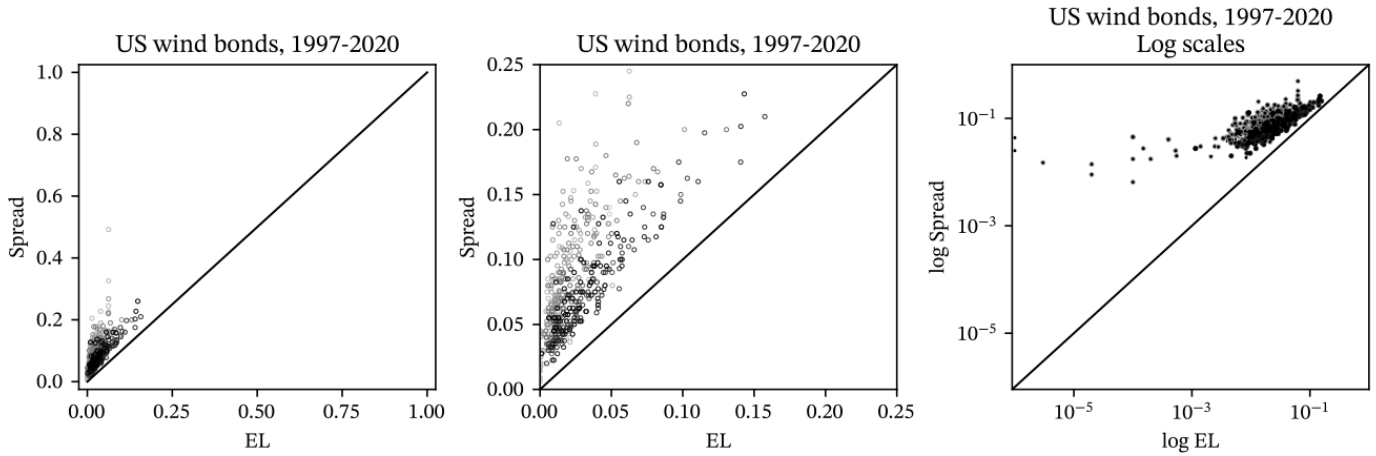

Figure 11.12 Spread (ROL) vs. EL on US wind (hurricane)-exposed catastrophe bonds since 1996. More recent years are shown darker black. The left and middle plots differ only in scale. Notice that catastrophe bond data includes observations for only $s < 0.20$. The right hand plot is on a log scale, emphasizing highly rated (low default probability) bonds, and illustrating the well-known minimum-rate-on line phenomenon of reinsurance pricing. Data: Lane Financial LLC.

Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley; Data Lane Financial, LLC

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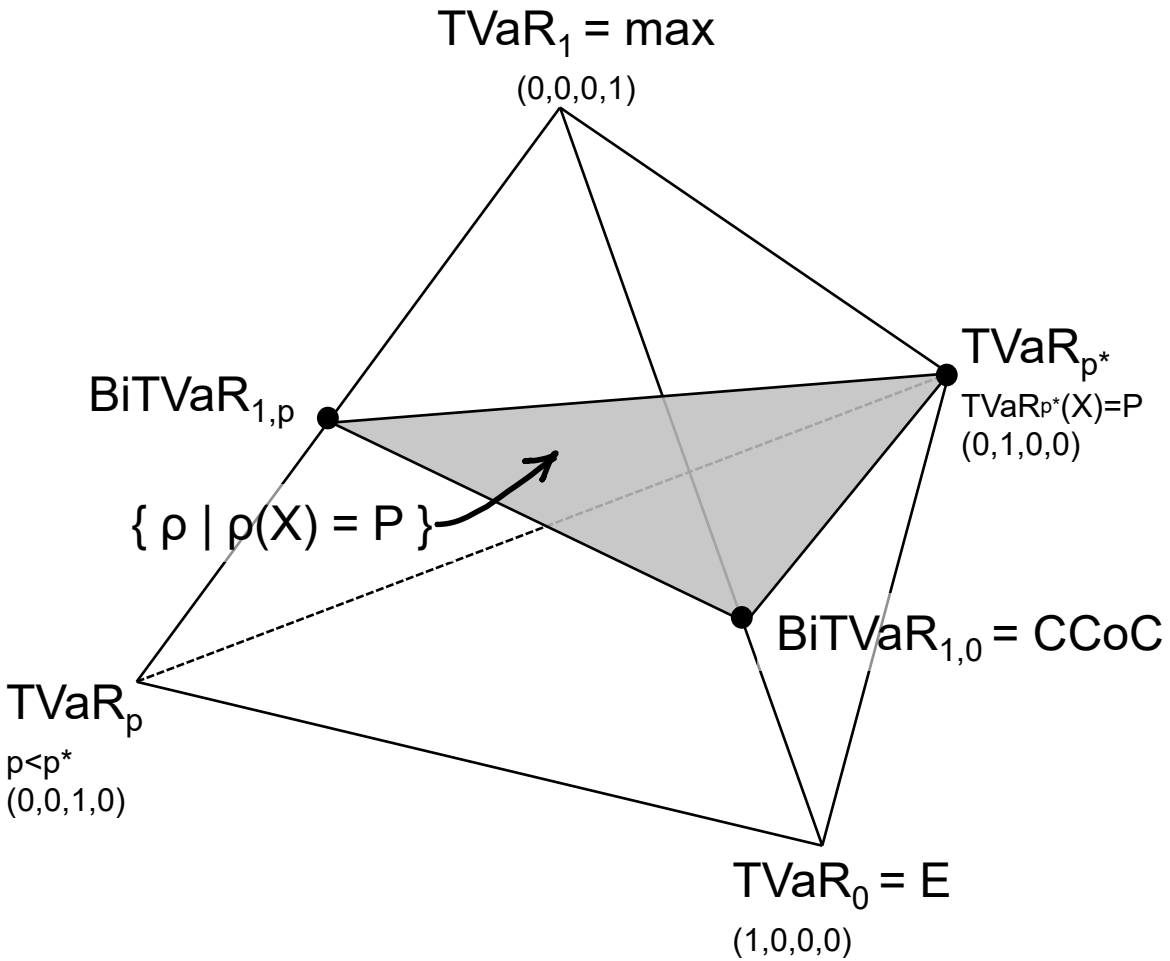
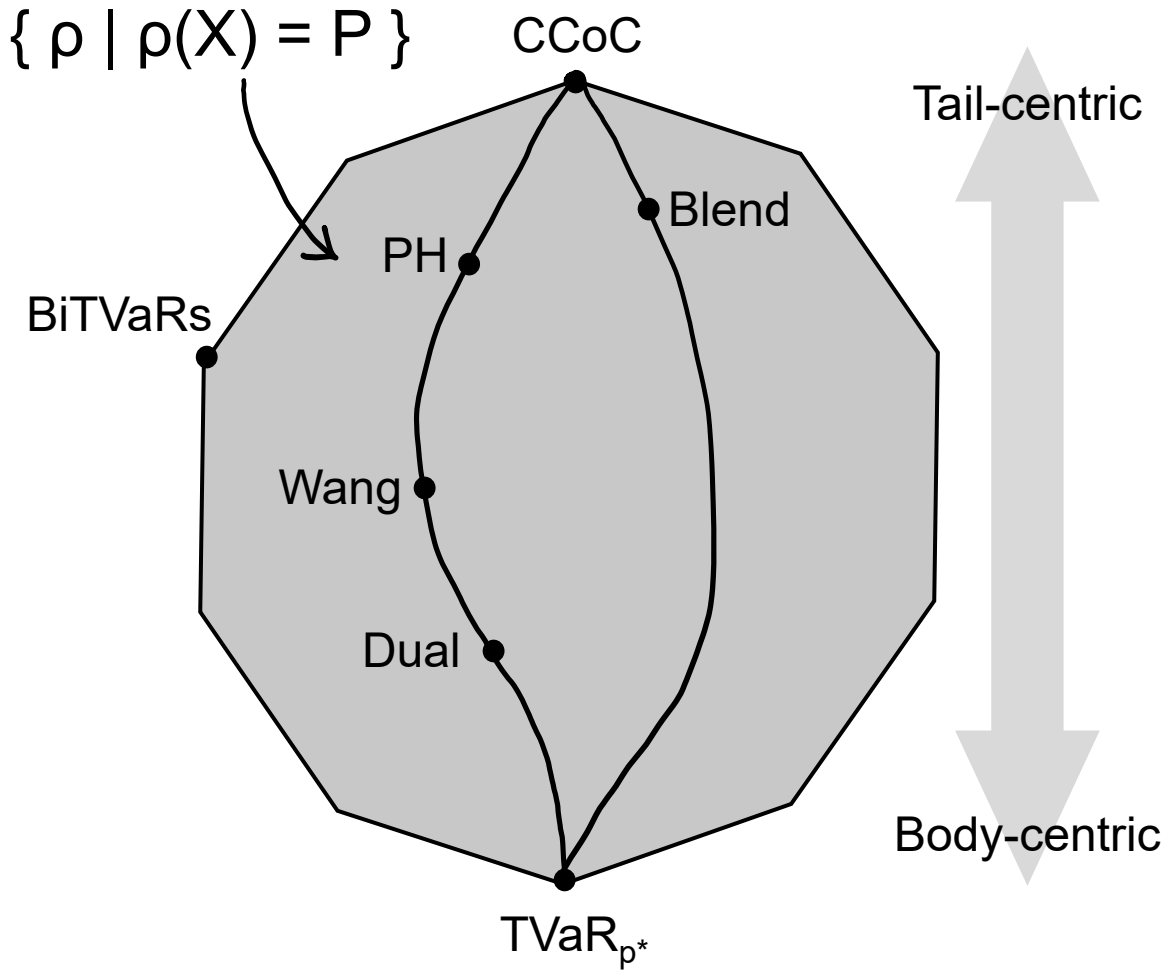
2b) Inferring g Indirectly: Determine Π = { ρ | ρ(X) = P }





Mildenhall – Similar risks have similar prices, IME 105 (2022): 203-210.

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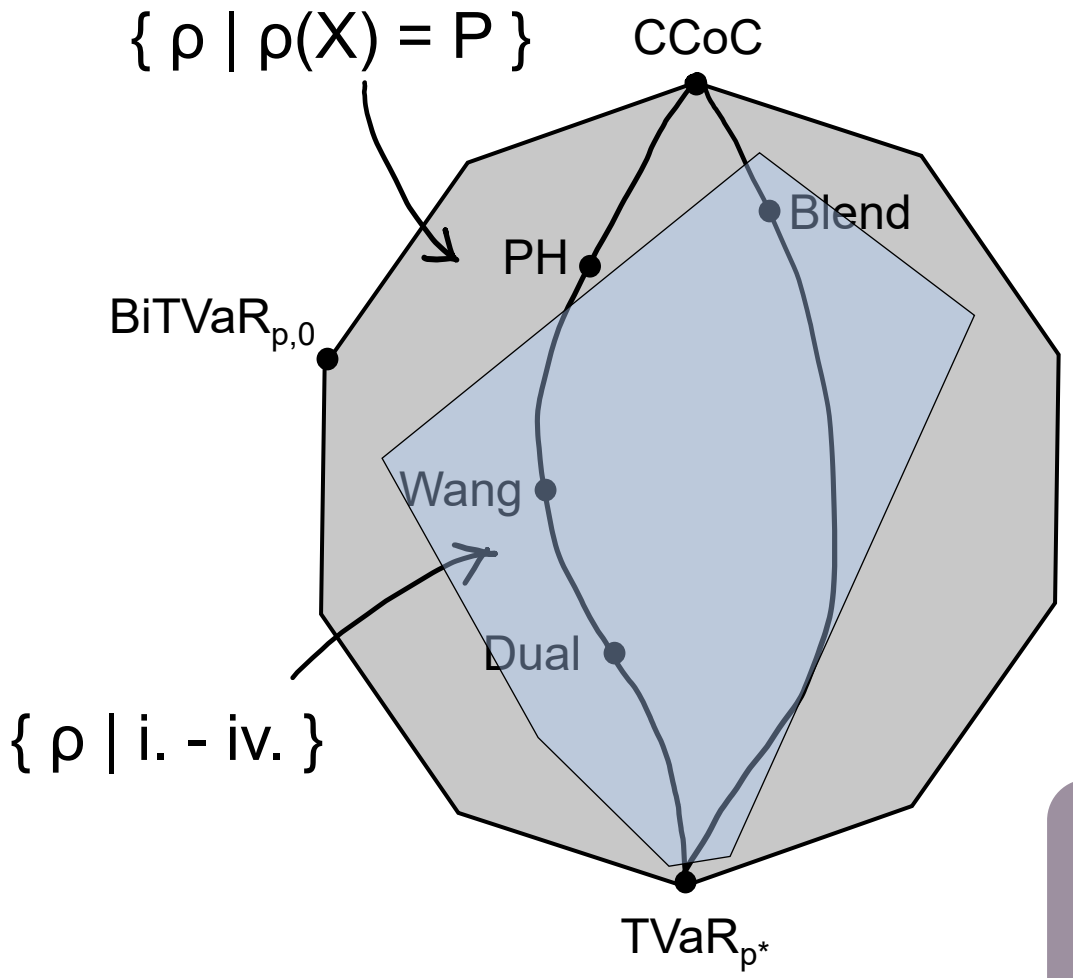
2) Determining g Directly and Indirectly



Constraints on g ↔ ρ

i. ρ(X) = P: total cost of capital
ii. g(s) < 1: equivalent measure
iii. Reproduces bond yields for small s
iv. Minimum rate-on-line behavior

Substantial reduction in size of Π

Parameterize to financing & strategic decisions as well as premiums.

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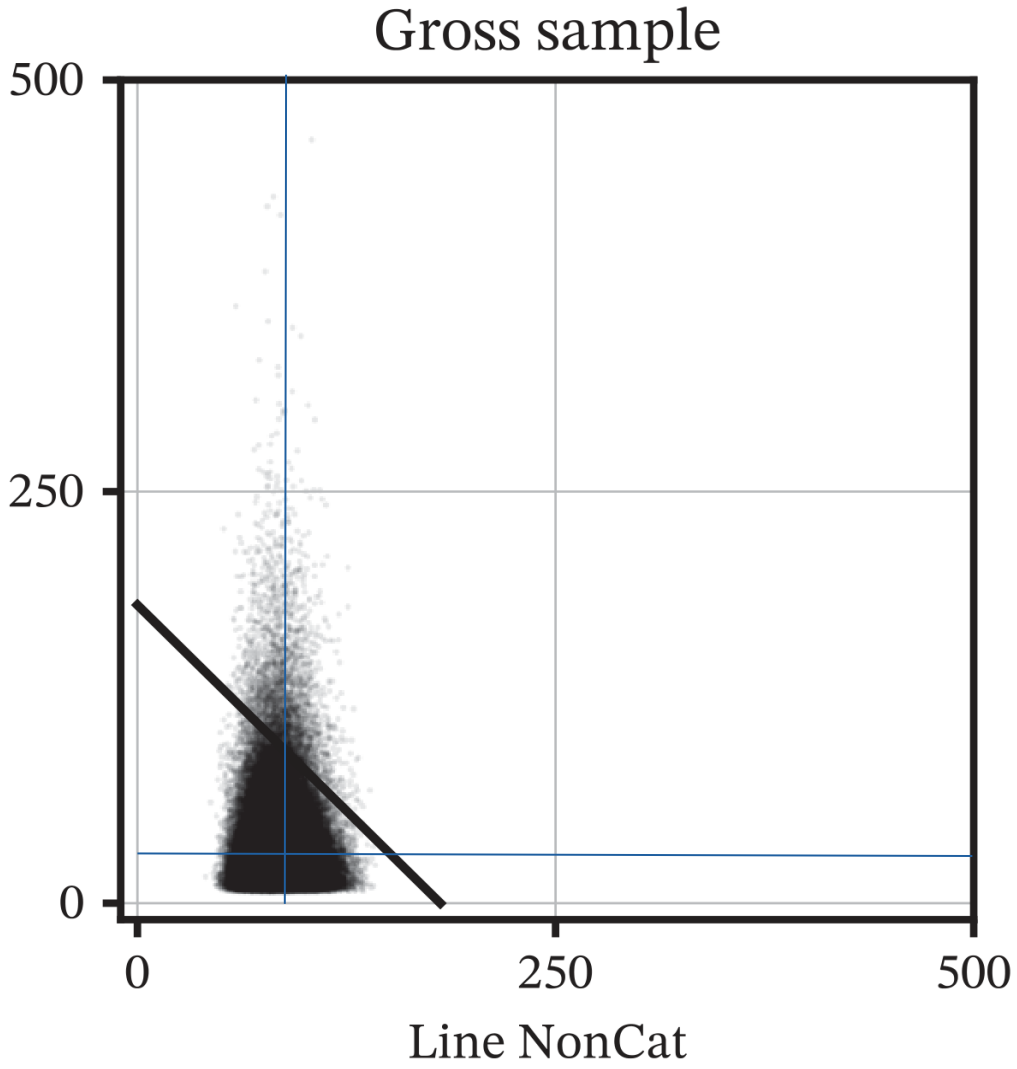
Cat/NonCat Case Study

Stochastic Model

- NonCat: gamma, mean 80, cv 0.15
- Cat: lognormal, mean 20, cv 1.0
- Independent
- Total: mean 100, cv 0.233

- 99.9% VaR asset requirement, 267.2
- Calibrate pricing to 115.15, the 10% CCoC pricing
- Scan QR code for full set of Case Study exhibits



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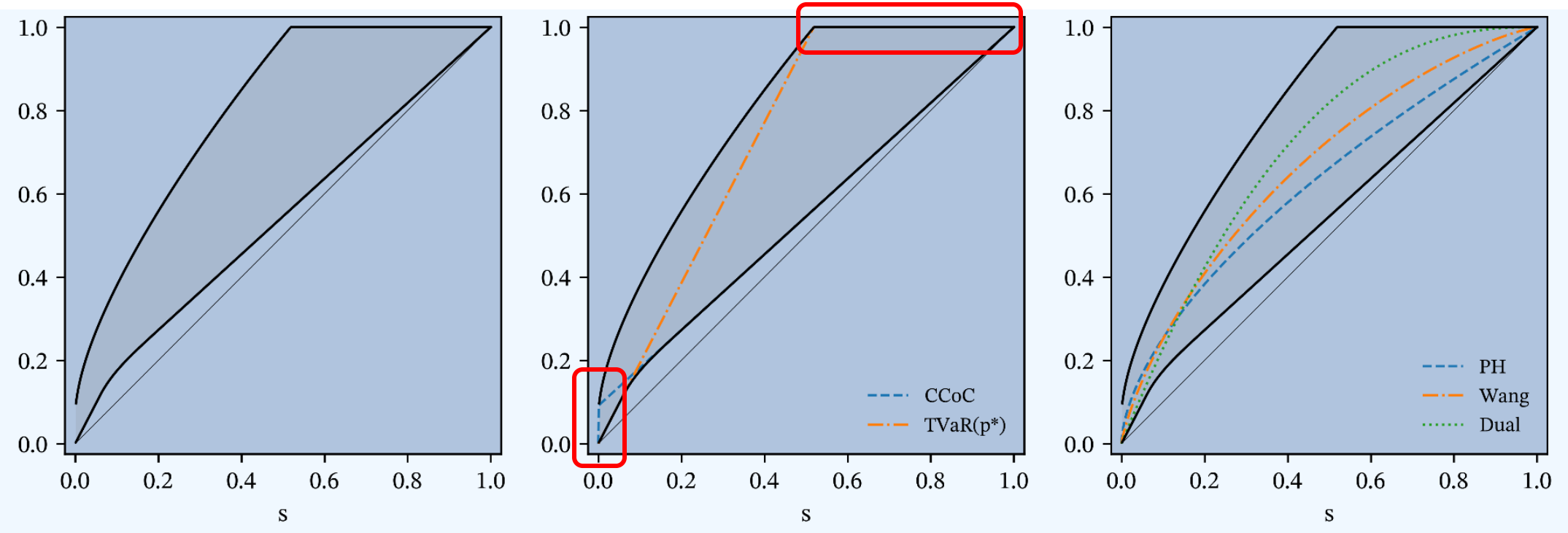
Distortion Envelope and Inferences about New Risks



- Any distortion in Π, pricing Cat/Non-Cat to P=115.15, lies in shaded region
- CCoC **tail-centric**: greatest as s → 0, most expensive tail capital
- $TVaR_{p^*}$ **body-centric**: greatest near 1, cheapest tail, most expensive body capital

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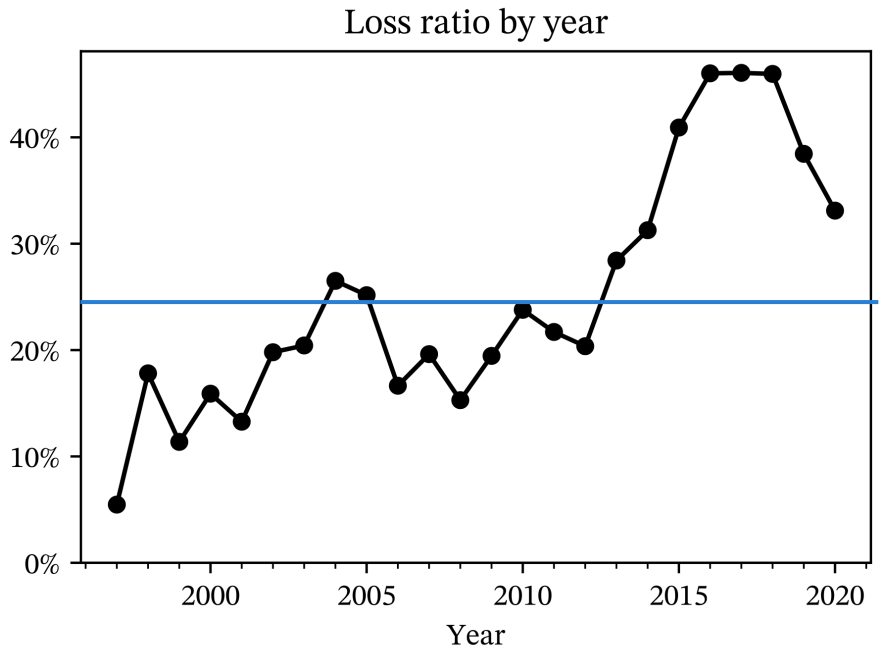
Distortion Envelope and Inferences about New Risks

For new risk Y, the extreme values of ρ(Y) over Π = { ρ | ρ(X) = P } occur at BiTVaRs, making them easy to compute

Application: evaluating reinsurance, Y = net

Cover: 80 x 41 aggregate, Cat line

- Minimum net premium of 105.90 occurs at ρ = CCoC, most tail-centric as expected
- Maximum of 110.88 occurs at ρ = $TVaR_{p^*}$
- Implied benefit from reinsurance ranges from 4.27 for $TVaR_{p^*}$ to 9.25 for CCoC
- Ceded loss = 2.22, ceded LR 24% to 52%
- **Range of outcomes brackets typical cat pricing: material to decisions!**



Average loss ratio by year 1997-2020, US wind exposed bonds only. Data: Lane Financial LLC.

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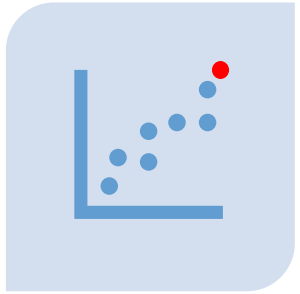
Conclusions

1. SRMs are a practical but under-specified pricing tool.

2. Parameterize to premium and financing data and decisions.

3. Firm's g encodes investor's view of bus-iness and management.



4. Firm's g influences risk appetite and reinsurance decisions.

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